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(6) ON CORRELATION SIGNAL DETECTION WITH PARTICULAR REFERENCE TO AN APPLICATION IN SONAR.

by

(10) M. J. Buckingham

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ON CORRELATION SIGNAL DETECTION WITH PARTICULAR REFERENCE TO AN
APPLICATION IN SONAR

by

M. J. Buckingham, PhD, CEng, MIEE

SUMMARY

Correlation detectors operate by exploiting the similarity between two
waveforms, one of which is the signal to be detected and the other of which is a
reference waveform. In practice the similarity occurs because both waveforms
will have been derived from the same source. As a measure of the similarity
between two signals we introduce here a 'similarity function' which is defined
in terms of the Fourier transforms of the signals rather than the signals them-
selves. An indication is given of the limited validity of this function as a
measure of similarity when the signals are of finite duration (as of course they
will be in an actual system).

An application of the similarity function is discussed in which a linear
FM sonar echo is bandpass-filtered prior to being correlated with a replica of
the transmitted signal. An analytical solution for the similarity function in
terms of the Q-factor of the filter is derived from which it can be seen that the
filter has little effect on the detection capability of the system provided that
the Q of the filter is less than the Q-factor of the linear FM pulse.

LIST OF CONTENTS

LIST OF SYMBOLS

t	time
j	$\sqrt{-1}$
f	frequency
$\omega = 2\pi f$	angular frequency
s	complex angular frequency
$f_1 = \omega_1/2\pi$	lowest frequency in the linear FM signal
$f_2 = \omega_2/2\pi$	highest frequency in the linear FM signal
$\Delta f = \Delta\omega/2\pi$	bandwidth of the linear FM signal
$f_0 = \omega_0/2\pi$	centre frequency of the linear FM signal
β	frequency-sweep-rate of the linear FM signal
$Q_p = \omega_0/\Delta\omega$	'Q-factor' of the linear FM signal
Q	quality factor of the bandpass filter
$S(\omega)$	power spectrum of the linear FM signal
T	duration of the linear FM signal

1   INTRODUCTION

Correlation detectors find application in situations where the signal to be detected (the echo) is derived in some way from some other signal, a replica of which is available as a reference. This causal connection between the replica and the echo suggests that the two waveforms should show some similar features. It does not mean that they are necessarily identical in form. A measure of the degree of similarity existing between the two signals is the entity which should be used as the detection criterion in correlation detection systems.

In section 2 of this paper our purpose is to examine the concept of similarity between two functions of time. Mathematically the approach to the problem is closely related to that taken by Gabor[1] in his "Theory of communication". This is perhaps not surprising since in both cases the aim is to make the best use of the information contained within a signal.

In the present case a 'similarity function' is defined which, unlike the cross-correlation function, provides in general *an optimum measure of similarity* between two signals. This similarity function and the ambiguity function discussed by Woodward[2] and subsequently used by various other authors including Kramer[3] and Harris and Kramer[4], are in fact the same in the special case where the echo is a Doppler-shifted version of the replica. The similarity function, however, has wider application than Woodward's ambiguity function and may be used quite generally, whatever the nature of the causal connection between the signals.

An example of how the similarity function may be used is discussed in section 3. The problem considered is encountered in sonar correlation-detection systems where the echo is bandpass-filtered prior to being correlated with a replica of the transmitted pulse. The effect of the filtering is to diminish the 'similarity' between the replica and the echo; that is, the spectral components of the two signals are no longer the same. We use the similarity function to determine as a function of the filter parameters, by how much the detection capability of the system is reduced as a result of the filtering process.

2   THE SIMILARITY FUNCTION

Let us consider two real functions of time, $x_1(t)$ and $x_2(t)$, of durations T_1 and T_2. That is, the amplitude of $x_1(t)$ takes non-zero values only within an interval of duration T_1, beyond which $x_1(t)$ is vanishingly small; and similarly, $x_2(t)$ takes finite values only in an interval of

duration T_2 . If the leading edges of the two functions occur at time $t = 0$, then

$$x_1(t) = [u(t) - u(t - T_1)]f_1(t) \tag{1}$$

$$x_2(t) = [u(t) - u(t - T_2)]f_2(t) \tag{2}$$

where $u(t)$ is the unit step function. And on delaying $x_2(t)$ with respect to $x_1(t)$ by a time τ we obtain the signal

$$x_2(t - \tau) = [u(t - \tau) - u(t - T_2 - \tau)]f_2(t - \tau) \quad . \tag{3}$$

If $\tau > T_1$ or $\tau < -T_2$ there is no overlap between $x_1(t)$ and $x_2(t - \tau)$. Otherwise, when the functions do overlap, the upper and the lower bounds, t_2 and t_1 , of the overlap interval depend on T_1, T_2 and τ in a way which may be determined by inspection of (1) and (3). The portions of the two pulses within the overlap region are given by the expressions

$$x_1'(t,\tau) = [u(t - t_1) - u(t - t_2)]f_1(t) \tag{4}$$

$$x_2'(t,\tau) = [u(t - t_1) - u(t - t_2)]f_2(t - \tau) \tag{5}$$

and only when $\tau = 0$ and $T_1 = T_2$ will the overlap functions in (4) and (5) be the same as the signals in (1) and (2).

We now introduce the following Fourier transforms:

$$X_1(j\omega) = \int_{-\infty}^{\infty} x_1(t)\exp - j\omega t dt = \int_{0}^{T_1} f_1(t)\exp - j\omega t dt \tag{6}$$

$$X_2(j\omega) = \int_{-\infty}^{\infty} x_2(t)\exp - j\omega t dt = \int_{0}^{T_2} f_2(t)\exp - j\omega t dt \tag{7}$$

which are just the transforms of the two signals $x_1(t)$ and $x_2(t)$ in the absence of any time delay. And the Fourier transforms of the two overlap functions in (4) and (5) are

$$X_1'(j\omega,\tau) = \int_{-\infty}^{\infty} x'(t,\tau)\exp - j\omega t dt = \int_{t_1}^{t_2} f_1(t)\exp - j\omega t dt \qquad (8)$$

and

$$X_2'(j\omega,\tau) = \int_{-\infty}^{\infty} x_2'(t,\tau)\exp - j\omega t dt = \int_{t_1}^{t_2} f_2(t - \tau)\exp - j\omega t dt . \qquad (9)$$

The transforms defined in equations (6) to (9) may be used to construct the function

$$\frac{\int_0^{\infty} X_1'(j\omega,\tau)X_2'^{*}(j\omega,\tau)d\omega}{\left[\int_0^{\infty} |X_1(j\omega)|^2 d\omega \int_0^{\infty} |X_2(j\omega)|^2 d\omega\right]^{\frac{1}{2}}} \qquad (10)$$

where the asterisk denotes complex conjugate and the integrals, it should be noted, are taken only over positive frequencies. The implications of excluding negative frequencies from the range of integration are discussed below.

Most practical correlation-detection systems will be designed so that overlap losses are negligibly small. In other words, the durations of the two pulses will be essentially the same, equal to T, say; and the value of τ at which a detection peak is observed will be very much less than T. Under these conditions, for all frequencies of interest, the transforms of $x_1(t)$ and $x_1'(t,\tau)$ are essentially the same; that is,

$$X_1'(j\omega,\tau) \simeq X_1(j\omega) \quad , \qquad (11)$$

and the transform of $x_2'(t,\tau)$ is given by the approximate relationship

$$X_2'(j\omega,\tau) \simeq X_2(j\omega)\exp - j\omega\tau \quad . \qquad (12)$$

Hence the expression in (10) may be approximated by the function

$$\Phi_{12}(\tau) = \frac{\int\limits_{0}^{\infty} X_1(j\omega)X_2^*(j\omega)\exp j\omega\tau d\omega}{\left[\int\limits_{0}^{\infty} |X_1(j\omega)|^2 d\omega \int\limits_{0}^{\infty} |X_2(j\omega)|^2 d\omega\right]^{\frac{1}{2}}} \quad . \tag{13}$$

We designate $\Phi_{12}(\tau)$ the similarity function between $x_1(t)$ and $x_2(t)$. And on a cautionary note we repeat that no account is taken in (13) of overlap losses. The formulation is valid only when $T_2 \simeq T_1 = T$ and $|\tau|/T \ll 1$.

The similarity function may be expressed either in terms of integrals over frequency, as it is in equation (13), or in terms of integrals over time. In order to go from the frequency domain to the time domain we follow Gabor[1] by defining $Z_i(j\omega)$ ($i = 1,2$) in terms of $X_i(j\omega)$ in such a way that all negative-frequency spectral components in the associated function of time, $z_i(t)$ are suppressed:

$$Z_i(j\omega) = 2X_i(j\omega)\left[u(\omega) - u(\omega - \omega_x)\right] \tag{14}$$

where $u(\omega)$ is the unit step function, and the right hand side of (14) is taken in the limit as ω_X goes to infinity. The function of time $z_i(t)$ is complex. It may be obtained from (14) by performing a simple convolution integration which yields

$$z_i(t) = x_i(t) + jy_i(t) \tag{15}$$

where

$$y_i(t) = \frac{1}{\pi} \int\limits_{-\infty}^{\infty} \frac{x_i(\nu)}{(t - \nu)} d\nu \quad . \tag{16}$$

The imaginary part of $z_i(t)$ is evidently not independent of $x_i(t)$, but is given by the Hilbert transform[5] of $x_i(t)$, as shown in (16).

The function $\Phi_{12}(\tau)$ may be expressed in terms of $Z_1(j\omega)$ and $Z_2(j\omega)$ in the form

$$\phi_{12}(\tau) = \frac{\displaystyle\int_{-\infty}^{\infty} Z_1(j\omega)Z_2^*(j\omega)\exp j\omega\tau d\omega}{\left[\displaystyle\int_{-\infty}^{\infty} |Z_1(j\omega)|^2 d\omega \int_{-\infty}^{\infty} |Z_2(j\omega)|^2 d\omega\right]^{\frac{1}{2}}} \tag{17}$$

where the limits of integration now extend from $+\infty$ to $-\infty$. Since Parseval's theorem states that

$$\int_{-\infty}^{\infty} g_1(t)g_2^*(t)dt = \int_{-\infty}^{\infty} G_1(j\omega)G_2^*(j\omega)df \tag{18}$$

where $g_i(t)$ and $G_i(j\omega)$ are a Fourier transform pair, we see that an alternative formulation to (17) is

$$\phi_{12}(\tau) = \frac{\displaystyle\int_{-\infty}^{\infty} z_1(t)z_2^*(t-\tau)dt}{\left[\displaystyle\int_{-\infty}^{\infty} |z_1(t)|^2 dt \int_{-\infty}^{\infty} |z_2(t)|^2 dt\right]^{\frac{1}{2}}}. \tag{19}$$

The real part of this expression is

$$Re\ \phi_{12}(\tau) = \frac{\displaystyle\int_{-\infty}^{\infty} x_1(t)x_2(t-\tau)dt}{\left[\displaystyle\int_{-\infty}^{\infty} x_1^2(t)dt \int_{-\infty}^{\infty} x_2^2(t)dt\right]^{\frac{1}{2}}} \tag{20}$$

which is just the normalized cross-correlation function between $x_1(t)$ and $x_2(t)$; and the imaginary part is

$$\text{Im } \phi_{12}(\tau) = \frac{\int\limits_{-\infty}^{\infty} \left\{ y_1(t)x_2(t - \tau) - x_1(t)y_2(t - \tau) \right\}dt}{2\left[\int\limits_{-\infty}^{\infty} x_1^2(t)dt \int\limits_{-\infty}^{\infty} x_2^2(t)dt \right]^{\frac{1}{2}}} \quad . \tag{21}$$

It is interesting to note that, had we started our analysis in the time domain, we should have chosen the correct function for the real part of $\phi_{12}(\tau)$; but it is unlikely that we should ever have selected the expression in (21) as the imaginary part of $\phi_{12}(\tau)$.

By the Schwarz inequality[6] the following is true:

$$\left. \begin{array}{l} 0 \leqslant \left| \phi_{12}(\tau) \right| \leqslant 1 \\[12pt] 0 \leqslant \left| \text{Re } \phi_{12}(\tau) \right| \leqslant 1 \\[12pt] 0 \leqslant \left| \text{Im } \phi_{12}(\tau) \right| \leqslant 1 \end{array} \right\} \tag{22}$$

and since in general the imaginary part of the similarity function is non-zero, it is also true that

$$\left| \phi_{12}(\tau) \right| \geqslant \left| \text{Re } \phi_{12}(\tau) \right| \quad . \tag{23}$$

The inequality in (23) indicates why the magnitude of similarity function rather than its real part (the cross-correlation function) should be used as the detection criterion. The difference between the two functions is essentially a difference in the degree of sensitivity each shows to phase information. This may be illustrated by considering as an example the case where the positive-frequency spectral components in $x_2(t)$ are all in quadrature with their counterparts in $x_1(t)$ and the relative amplitudes of the spectral components are all frequency independent. The relationship between the transforms of $x_1(t)$ and $x_2(t)$ is then

$$\left. \begin{array}{ll} X_2(j\omega) = jX_1(j\omega) & \text{for all } \omega \geqslant 0 \\[12pt] X_2(j\omega) = -jX_1(j\omega) & \text{for all } \omega < 0 \end{array} \right\} \tag{24}$$

and hence

$$\Phi_{12}(0) = -j \quad ; \qquad |\Phi_{12}(0)| = 1 \quad . \tag{25}$$

On the other hand, we have

$$\text{Re } \Phi_{12}(\tau) = 0 \qquad \text{for all } \tau \quad . \tag{26}$$

Clearly, the similarity function shows more tolerance towards phase differences between the spectral components of $x_1(t)$ and $x_2(t)$ than does the cross-correlation function.

The similarity function takes a particularly simple form when $x_1(t)$ and $x_2(t)$ are the input and output respectively of a linear system. If the system function is $H(j\omega)$ then the relationship between the Fourier transforms of the two signals is

$$X_2(j\omega) = H(j\omega)X_1(j\omega) \tag{27}$$

and hence from (13)

$$\Phi_{12}(\tau) = \frac{\displaystyle\int_0^\infty H^*(j\omega)|X_1(j\omega)|^2\exp j\omega\tau d\omega}{\left[\displaystyle\int_0^\infty |X_1(j\omega)|^2 d\omega \int_0^\infty |H(j\omega)|^2|X_1(j\omega)|^2 d\omega\right]^{\frac{1}{2}}} \quad . \tag{28}$$

Assuming that the system function is known, it is necessary only to know in addition the power spectrum of the input signal in order to be able to evaluate this expression. In the special case where the system function is independent of frequency and τ is zero, not even the power spectrum is required, since the similarity function is simply equal to $H^*(j\omega)/|H(j\omega)|$ and $|\Phi_{12}(0)| = 1$. We shall make use of the formulation of the similarity function given by (28) in the following section.

3   A PROBLEM IN UNDERWATER ACOUSTIC SIGNAL DETECTION

In modern sonar systems, linearly-swept FM signals are used for detecting objects whose radial velocity in relation to the detector is too low to shift (due to the Doppler effect) the spectral components of the echo away from the

frequency region in which spurious signals caused by reverberation predominate. Compared with its radar counterpart, the sonar FM signal has a high time-bandwidth product, usually taking a value in the region of several hundred. The implications of this as far as the detection of a Doppler-shifted echo is concerned have been discussed by various authors but most notably by Kramer[4] whose analysis lays the foundation of the subject. The problem we discuss below is also concerned with the detection of a distorted signal, but in this case the echo is a bandpass-filtered version of the linear FM replica. The discussion is of more than academic interest since in actual sonar systems it is likely that the echo will indeed be filtered in order to remove the spectral components of the noise lying outside the frequency band of interest.

In order to avoid significant overlap losses the relaxation time of the filter must be very much less than the duration of the transmitted pulse. For most correlation sonars this condition would be satisfied if the Q-factor of the filter were less than about 100. As the Q of the filter will be very much less than this in practice (usually in the region of 20) in order to maintain an adequately high detection peak, we are wholly justified in assuming that the durations of the replica and of the bandpass-filtered echo are effectively the same. Provided that the time delay between the replica and the echo is very much less than the duration of the two signals, the form of the similarity function given in (28) is therefore a valid measure for establishing the presence of the echo.

We make the further assumptions in the analysis that the echo has been reflected from a stationary point object; and that the centre frequencies of the FM pulse and the pass-band of the filter are the same. The latter assumption is not essential to the argument, but since it leads to a simplification of the algebra we use it here in order to illustrate the method.

Let us suppose that the transmitted linear FM pulse has a centre frequency $\omega_0/2\pi$, a frequency-sweep-rate β and duration T . If the function $x_1(t)$ of the previous section is identified as a replica of this pulse, then

$$x_1(t) = \left[u(t) - u(t - T)\right] f_1(t) \tag{29}$$

where $\qquad f_1(t) = \sin\left\{\phi(t)\right\}$; $\qquad \dfrac{d\phi(t)}{dt} = \omega_0\left[1 + \beta\left(t - \dfrac{T}{2}\right)\right]$. $\qquad$ (30)

From these expressions the frequency-sweep-rate of the pulse is

$$\beta = \frac{\omega_2 - \omega_1}{\omega_0 T} = \frac{\Delta\omega}{\omega_0 T} \tag{31}$$

where $\omega_2/2\pi$ and $\omega_1/2\pi$ are the highest and lowest frequencies in the pulse, and $\Delta\omega/2\pi$ is the pulse bandwidth.

In order to evaluate (28), the power spectrum of $x_1(t)$ is required. Fig 1 shows an example of a computed spectrum for frequencies close to the centre frequency of the replica. Perhaps the most striking feature of this curve is its essentially rectangular character. For our present purpose we ignore the structure on the curve and approximate the spectrum around $\omega = \omega_0$ by a truly rectangular distribution. That is, we put

$$S(\omega) \equiv \frac{2}{T} |X_1(j\omega)|^2 \simeq \frac{\pi}{\Delta\omega} \left[u(\omega - \omega_1) - u(\omega - \omega_2) \right] \tag{32}$$

where $S(\omega)$ is the power spectrum of the FM pulse, and the multiplying constant, $\pi/\Delta\omega$, on the right of (32) appears in order to maintain consistency with equations (29) and (30).

The approximation in (32) should be adequate for evaluating the integrals in the similarity function. For, if the structure on the spectral density curve were not neglected, its effect on these integrals would more or less average out since a large number of oscillations occur within the range of integration (that is, over the bandwidth of the pulse).

By (32) the similarity function in (28) is now

$$\phi_{12}(\tau) = \frac{\dfrac{1}{\sqrt{\Delta\omega}} \displaystyle\int_{\omega_1}^{\omega_2} H^*(j\omega)\exp j\omega\tau d\omega}{\left[\displaystyle\int_{\omega_1}^{\omega_2} |H(j\omega)|^2 d\omega \right]^{\frac{1}{2}}} \ . \tag{33}$$

For a symmetrical response about $\omega = \omega_0$ we choose the system function in (33) to be

$$H(j\omega) = \frac{1}{1 + 2jQ \dfrac{(\omega - \omega_0)}{\omega_0}} \quad . \tag{34}$$

The term on the right of (34) is the Lorentz function. Strictly a second Lorentz function should be included in (34) to ensure that $H(j\omega)$ shows conjugate symmetry. However, for frequencies in the region of $\omega = \omega_0$, this second term is negligibly small and so, for our purpose, (34) is acceptable.

The real and imaginary parts of the system function are

$$H_R(\rho) = \frac{1}{1 + 4Q^2\rho^2} \quad ; \qquad H_I(\rho) = \frac{-2Q\rho}{1 + 4Q^2\rho^2} \tag{35}$$

where we have introduced the new variable

$$\rho = \frac{(\omega - \omega_0)}{\omega_0} \quad . \tag{36}$$

On recognizing that $H_R(\rho)$ and $H_I(\rho)$ are, respectively, even and odd functions of ρ , we see that $|\phi_{12}(\tau)|$ from (33) reduces to the form

$$|\phi_{12}(\tau)| = \frac{Q_p^{\frac{1}{2}} \displaystyle\int_{-\frac{1}{2Q_p}}^{\frac{1}{2Q_p}} \left\{ H_R(\rho) \cos \omega_0 \rho \tau + H_I(\rho) \sin \omega_0 \rho \tau \right\} d\rho}{\left[\displaystyle\int_{-\frac{1}{2Q_p}}^{\frac{1}{2Q_p}} \left\{ H_R^2(\rho) + H_I^2(\rho) \right\} d\rho \right]^{\frac{1}{2}}} \tag{37}$$

where $Q_p = \omega_0/\Delta\omega$ is the 'Q-factor' of the pulse. In order to estimate the magnitude of the principal maximum in this expression we put $\tau = 0$. In fact this is not an optimum condition when the Q of the filter is larger than Q_p . The result we obtain is therefore a 'worst case' estimate of system performance which will not necessarily be observed in practice.

For zero time delay between the replica and the echo, (37) becomes

$$|\Phi_{12}(0)| = \left[Q_p \int_{-\frac{1}{2Q_p}}^{\frac{1}{2Q_p}} \frac{d\rho}{(1 + 4Q^2\rho^2)} \right]^{\frac{1}{2}}$$

$$= \left[\left(\frac{Q_p}{Q} \right) \tan^{-1} \left(\frac{Q}{Q_p} \right) \right]^{\frac{1}{2}} . \tag{38}$$

This expression is plotted in Fig 2 as a function of Q/Q_p . From the shape of this function it is clear that in practice it is desirable to choose the Q-factor of the filter to be no greater than the Q of the pulse. This is not an entirely unexpected result since it is when Q becomes larger than Q_p that the spectral components of the echo in the regions of $\omega = \omega_1$ and $\omega = \omega_2$ start to suffer significant phase changes as a result of the filtering.

The analytical method described above is valid for any system function provided that the system to which it relates is linear and, as already stated, that any relaxation time associated with the system is short compared with the duration of the replica. For the large class of systems which can be described by a linear differential equation with constant coefficients, $H(s)$ is a rational function of s ; that is, $H(s)$ is expressible as the ratio of two polynomials in s . In most cases of interest the order of the denominator will be higher than that of the numerator (the system function given in equation (34) falls into this category); in other words the output of the system will fall to zero as the frequency approaches infinity. If $H(s)$ is such a function, then, for $\tau = 0$ at least, the two integrals in (28) involving the system function can always be evaluated analytically (assuming that (32) is used for approximating the power spectrum of the replica), and in general the result for each will be the sum of a number of logarithmic functions, a number of inverse tangents and a number of rational functions of the type $A/\{(\lambda - 1)(\omega - p)^{\lambda-1}\}$, where λ is an integer greater than 1.

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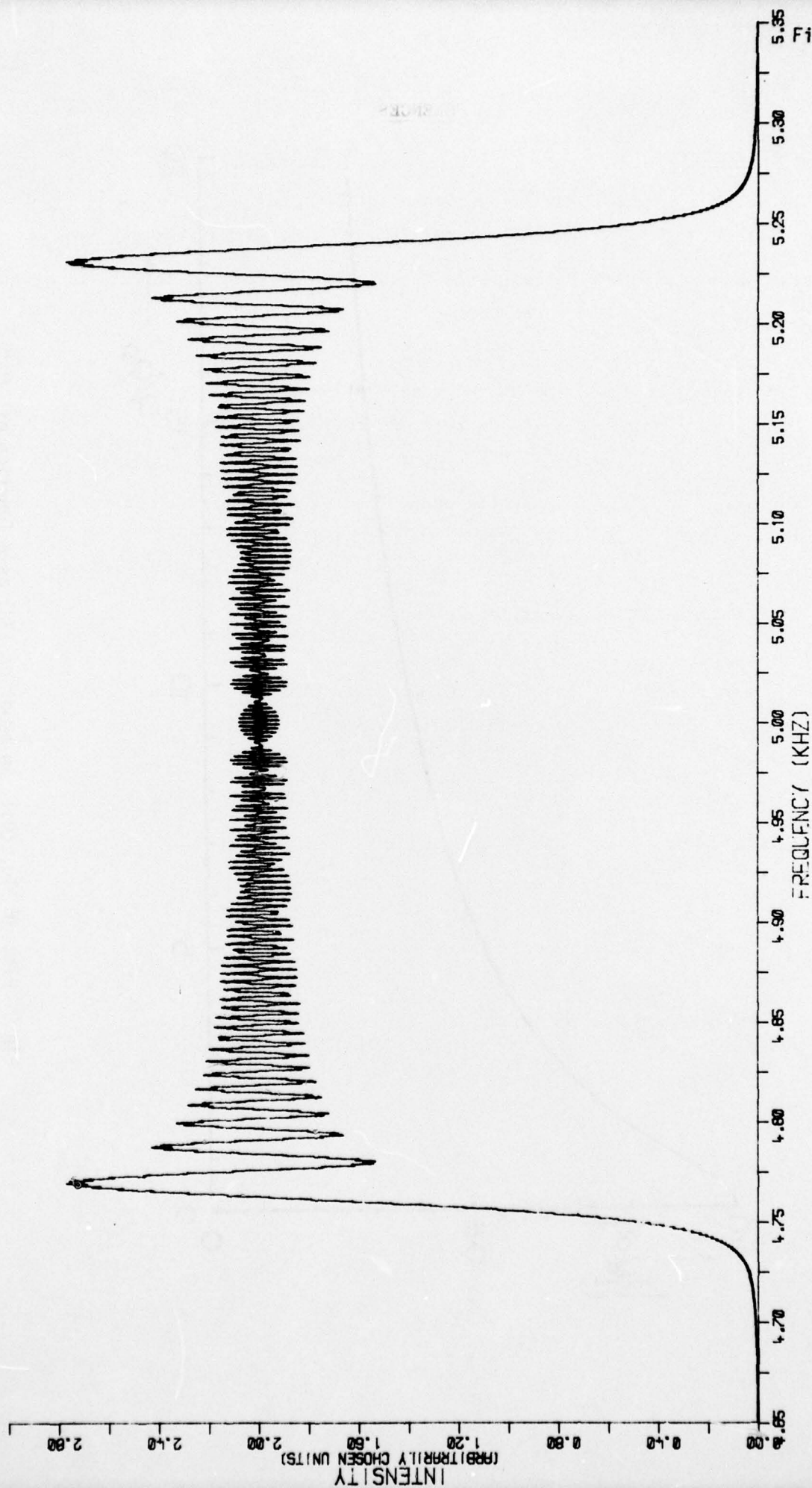
Fig 1 Power spectrum of the linear FM pulse. The signal parameters are: $\Delta f = 500$ Hz, $f_0 = 5$ kHz and $T = 1$ second
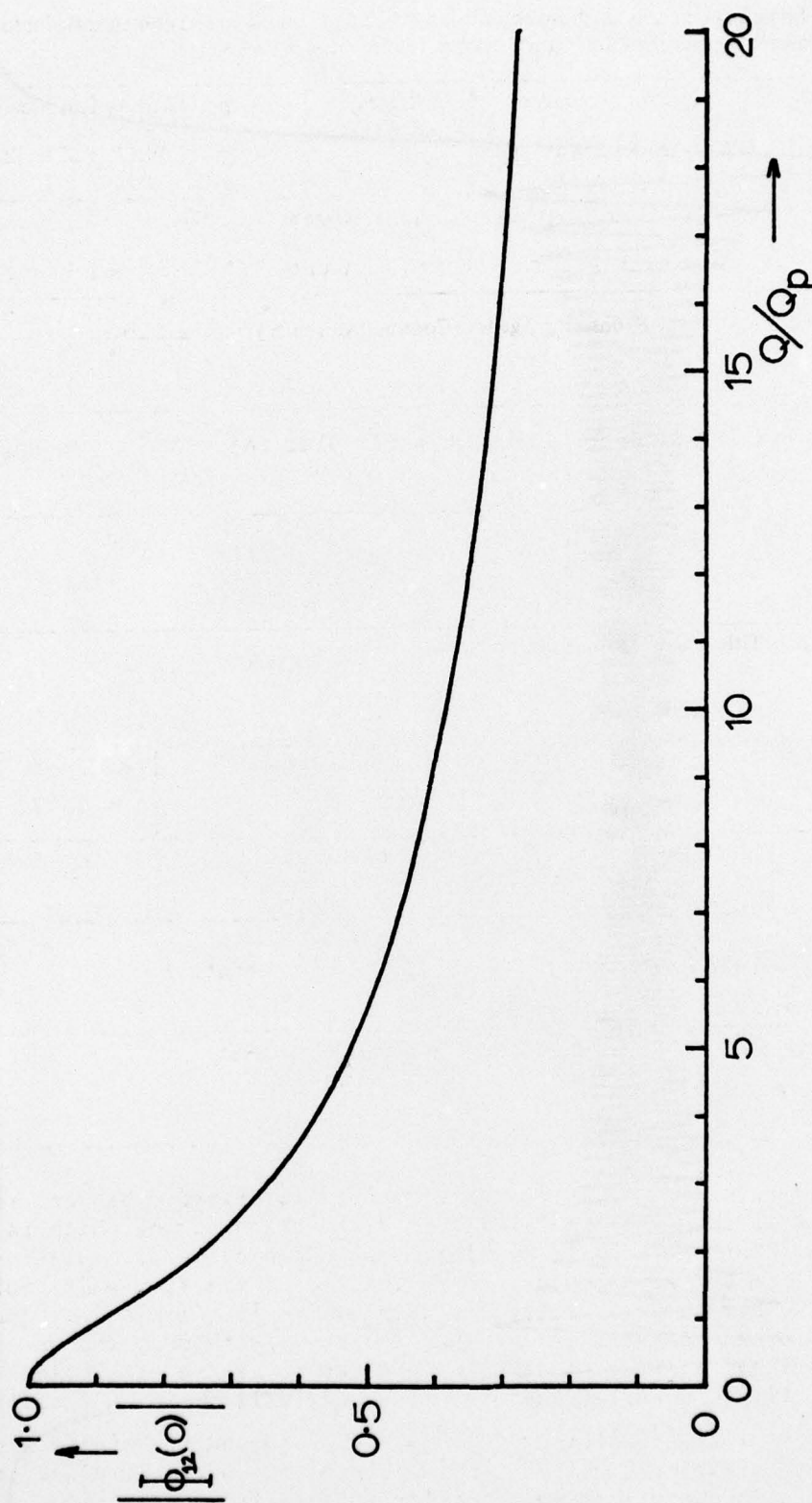
Fig 2

Fig 2 Plot of $|\Phi_{12}(0)|$ in equation (38) as a function of (Q/Q_p)

REPORT DOCUMENTATION PAGE

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17. Abstract

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An application of the similarity function is discussed in which a linear FM sonar echo is bandpass-filtered prior to being correlated with a replica of the transmitted signal. An analytical solution for the similarity function in terms of the Q-factor of the filter is derived from which it can be seen that the filter has little effect on the detection capability of the system provided that the Q of the filter is less than the Q-factor of the linear FM pulse.

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